Trigonometry 3

1. Express $\sin 2\theta + \sqrt{3}\cos 2\theta$ in the form $R\sin(2\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

   Hence,

   (i) find the maximum value of $\left(\sin 2\theta + \sqrt{3}\cos 2\theta\right)^2$ and the corresponding angle, θ at which the maximum value occurs.

   (ii) Solve $\sin 2\theta + \sqrt{3}\cos 2\theta = 1$ for $0 \le \theta \le \pi$.

2. Prove **(i)** $cos^{-1}(-x) = \pi - cos^{-1}x$ **(ii)** $\frac{1}{4}\pi + tan^{-1}x = tan^{-1}\left(\frac{1+x}{1-x}\right)$

3. Prove : $\frac{cos^4\theta + sin^4\theta}{cos^4\theta - sin^4\theta} = \frac{1}{2}(\cos 2\theta + \sec 2\theta)$

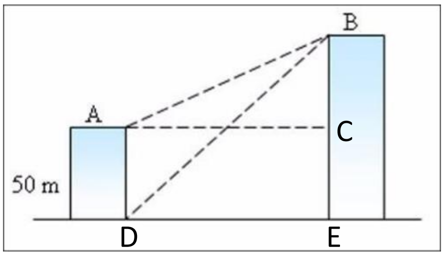
4. Solve the equation $2\cos\theta + 5\sin\theta = 4$ by expressing in the form $cos\,(2\theta + \alpha)$, where $0° \le \theta \le 360°$

5. Prove that : $\frac{d^n}{dx^n}\sin x = sin\left(x + \frac{n\pi}{2}\right)$

6. **(i)** Find the set of values of x in the interval $-\pi < x < \pi$ such that $|\sin 2x| > \frac{1}{2}$.

   (ii) From **(i)**, find the set of values of x in the interval $-\pi < x < \pi$ such that $|\sin 2x| > \frac{1}{2}$

7. Referring to the diagram, building A is measured with 50 m in height. The angle from the base of the building A to the highest point of building B is measured as $62°$. The angle formed from the rooftop of building A right to the highest point of building B is $59°$. What is the distance that keeps the two buildings apart?



8. It is given that $f(x) = 11\,cos^2x - 6\sin x\cos x + 3\,sin^2x$.

   (a) Express $f(x)$ in the form of $a\,cos\,2x + b\sin 2x + c$, where a, b and c are constants.

   (b) Show that $f(x)$ can be expressed in the form of $rcos\,(2x + \alpha) + c$, where r and c are constants and $\tan\alpha = \frac{3}{4}$.

   (c) Find the maximum and minimum values of the express $\frac{1}{f(x)}$.

   (d) Find the values of x between $0°$ and $180°$ such that $f(x) = 8$.

   (e) Find the set of values of x in the interval $0° \le x \le 180°$ such that $2 \le f(x) \le 4.5$